Lett Spat Resour Sci (2011) 4: 139–156
DOI 10.1007/s12076-011-0058-8

ORIGINAL PAPER

A stepwise-projection data envelopment analysis for public transport operations in Japan

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Received: 15 July 2010 / Accepted: 19 April 2011 / Published online: 3 May 2011

Abstract Tightening public expenditure budgets prompt a need for a careful analysis of the performance of public bodies in terms of an efficient execution of their tasks. A standard tool to judge the efficiency of such organizations is Data Envelopment Analysis (DEA). In the past years, much progress has been made to extend this approach into various directions. Examples are the Distance Friction Minimization (DFM) model and the Context-Dependent (CD) model.

The DFM model is based on a generalized distance friction function and serves to improve the performance of a Decision Making Unit (DMU) by identifying the most appropriate movement towards the efficiency frontier surface. Likewise, the CD model yields efficient frontiers in different levels, while it is based on a level-by-level improvement projection.

The present paper will first offer a new integrated DEA tool—emerging from a blend of the DFM and CD model—in order to design a balanced stepwise efficiency-improving projection model for a conventional DEA. The above-mentioned stepwise-projection model is illustrated on the basis of an application to the efficiency analysis of public transport operations in Japan.

Keywords Data Envelopment Analysis (DEA) · Stepwise projection · Distance friction minimization · Context-dependence · Public transport operations

JEL Classification R40 · C44

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1 Introduction

With tightening budgets and increasingly critical reviews of public expenditure, there is a need for a careful analysis of the performance of public bodies in terms of an efficient execution of their tasks. These questions show up everywhere in the public domain, for instance, in the provision of medical facilities, the operation of postal services, or the supply of public transport. The need for a critical efficiency judgment of public agencies may also stem from exogenous circumstances; for example, the rapid ageing process in Japan calls for a careful analysis of the performance of public transport facilities.

A standard tool to judge the efficiency of such agencies is Data Envelopment Analysis (DEA). DEA has gained much importance in economic performance studies. Seiford (2005) mentions some 2800 published articles on DEA. This large number of studies shows that comparative efficiency analysis has become an important instrument for a benchmark analysis in both the private and public sector. DEA was developed to analyze the relative efficiency of a Decision Making Unit (DMU), by constructing a piecewise linear production frontier and by mapping out the relative position of the DMU concerned. A DMU that is located on the frontier is efficient, while a DMU that is not on the frontier is inefficient. An inefficient DMU may become more efficient by reducing its inputs or increasing its outputs. In the standard DEA approach, this is achieved by a uniform reduction in all inputs (or a uniform increase in all outputs). But in principle, there is an infinite number of improvements in order to reach the efficient frontier, and hence there are many solutions for a DMU to enhance efficiency. The existence of an infinite number of solutions to reach the efficient frontier has led to a stream of literature on the integration of DEA and Multiple Objective Linear Programming (MOLP), which was initiated by Golany (1988).

Suzuki et al. (2010, 2011) proposed a Distance Friction Minimization (DFM) model that is based on a generalized distance friction function and serves to improve the performance of a DMU by identifying the most appropriate movement towards the efficiency frontier surface. This approach may address both an input reduction and an output increase as a strategy of a DMU. A suitable form of multidimensional projection functions is given by a Multiple Objective Quadratic Programming (MOQP) model using a Euclidean distance. A general efficiency-improving projection model including a DFM model is able to calculate either an optimal input reduction value or an output increase value to reach a full efficiency score of 1, even though in reality this may be hard to achieve. For example, it may be nearly impossible for one single regional city public transport system (e.g., Kyoto transportation authorities) to completely attain a maximum efficiency with one metropolitan private transport company (e.g., Tokyo METRO). It is therefore meaningful to develop a more practical and feasible efficiency improving projection model, especially based on the DFM model.

It is noteworthy that Seiford and Zhu (2003) developed a gradual improvement model for an inefficient DMU. This 'Context-Dependent (CD)' DEA has an important merit, as it aims to reach a stepwise improvement through successive levels towards the efficiency frontier. The CD model will be used as an ingredient in our DFM model.

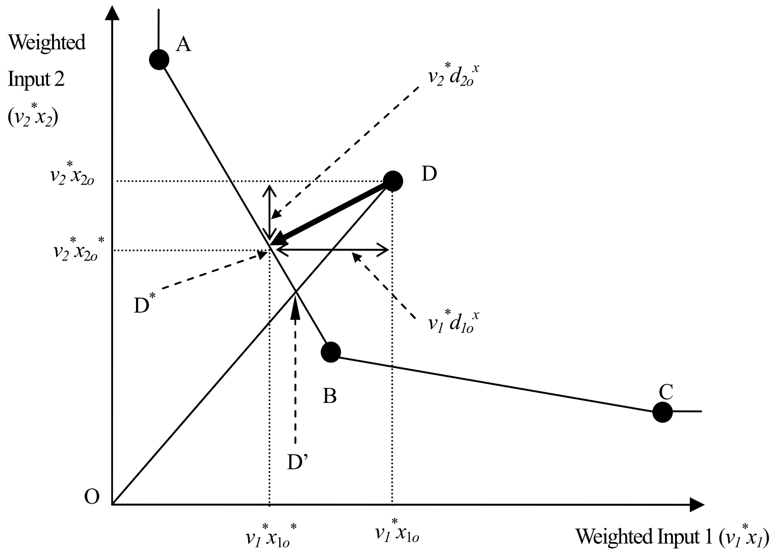


Fig. 1 Illustration of the DFM approach (input—$v_i^* x_i$ space)

This paper aims to develop a new integrated DEA tool emerging from a blend of the DFM and CD model, namely a Stepwise DFM model. This model can provide a stepwise efficiency-improving projection to provide more practical and feasible solutions for realistic circumstances and requirements in an efficiency-improving projection. The above mentioned stepwise-projection model is applied to a performance analysis in the context of an efficiency-improving plan for public transport operations in Japan.

2 The Distance Friction Minimization (DFM) approach

An efficiency improvement solution in the original CCR-input model (see Appendix) requires that the input values are reduced radially by a uniform ratio θ^* ($\theta^* =$ OD'/OD, see Appendix, Fig. 9). The (v^*, u^*) values obtained as an optimal solution for formula (A.1) result in a set of optimal weights for $\mathrm{DMU_o}$.

As explained in Appendix, (v^*, u^*) is the set of most favourable weights for $\mathrm{DMU_o}$, in the sense of maximizing the ratio scale. Thus, v_m^* is the optimal weight for the input item m, and its magnitude expresses how much in relative terms the item is contributing to efficiency. Similarly, u_s^* does the same for the output item s. These values show not only which items contribute to the performance of $\mathrm{DMU_o}$, but also to what extent they do so. In other words, it is possible to express the distance frictions (or alternatively, the potential increases) in improvement projections.

In this study, we use the optimal weights u_s^* and v_m^* from (A.1), and then describe next the efficiency improvement projection model. A visual presentation of this new approach is given in Figs. 1 and 2.

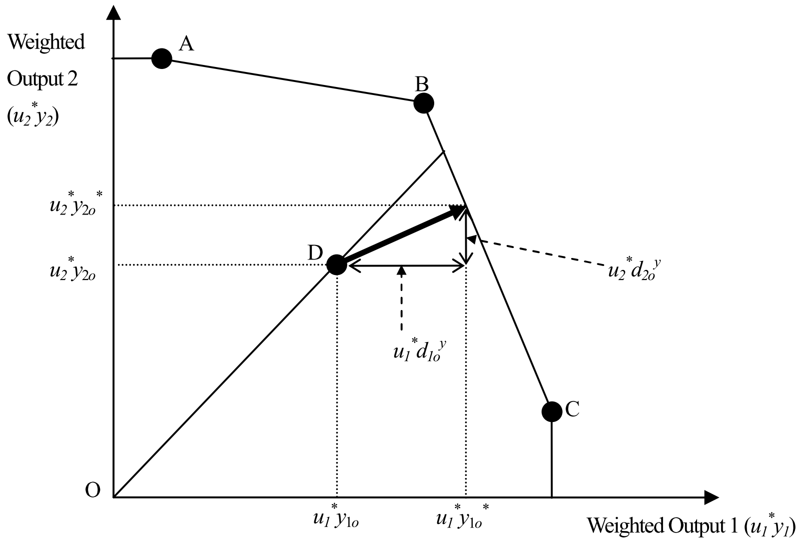


Fig. 2 Illustration of the DFM approach (output—$u_r^* y_r$ space)

In this approach a generalized distance friction is deployed to assist a DMU in improving its efficiency by a movement towards the efficiency frontier surface. The direction of efficiency improvement depends of course on the input/output data characteristics of the DMU. It is now appropriate to define the projection functions for the minimization of distance friction by using a Euclidean distance in weighted spaces. As mentioned, a suitable form of multidimensional projection functions that serves to improve efficiency is given by a MOQP model which aims to minimize the aggregated input reduction frictions, as well as the aggregated output increase frictions. Thus, the DFM approach can generate a new contribution to efficiency enhancement problems in decision analysis, by deploying a weighted Euclidean projection function, and at the same time it may address both input reduction and output increase. The details of this approach have been outlined elsewhere (see Suzuki et al. 2010, 2011). Here we will only describe the various steps concisely.

First, specify the distance friction function Fr^x and Fr^y by means of (2.1) and (2.2), which are defined by the Euclidean distance shown in Figs. 1 and 2. Next, solve the following MOQP by using d_{mo}^x (a reduction of distance for x_{io}) and d_{so}^y (an increase of distance for y_{so}) as minimands in an L_2 metric:

$$\min\ Fr^x = \sqrt{\sum_m (v_m^* x_{mo} - v_m^* d_{mo}^x)^2} \tag{2.1}$$

$$\min\ Fr^y = \sqrt{\sum_s (u_s^* y_{so} - u_s^* d_{so}^y)^2} \tag{2.2}$$

$$\text{s.t.} \quad \sum_m v_m^* (x_{mo} - d_{mo}^x) = \frac{2\theta^*}{1+\theta^*} \tag{2.3}$$

$$\sum_s u_s^*(y_{so} + d_{so}^y) = \frac{2\theta^*}{1+\theta^*} \tag{2.4}$$

$$x_{mo} - d_{mo}^x \geq 0 \tag{2.5}$$

$$d_{mo}^x \geq 0 \tag{2.6}$$

$$d_{so}^y \geq 0, \tag{2.7}$$

where x_{mo} is the amount of input item m for any arbitrary inefficient DMU_o, and y_{so} is the amount of output item s for any arbitrary inefficient DMU_o. The constraint functions (2.3) and (2.4) refer to the target values of input reduction and output increase. A balanced distribution of contributions from the input and output side to achieve efficiency is established as follows. The total efficiency gap to be covered by inputs and outputs is $(1 - \theta^*)$. The input and the output side contribute according to their initial levels 1 and θ^*, implying shares $\theta^*/(1 + \theta^*)$ and $1/(1 + \theta^*)$ in the improvement contribution. Hence, the contributions from both sides equal $(1 - \theta^*)[\theta^*/(1 + \theta^*)]$ and $(1 - \theta^*)[1/(1 + \theta^*)]$, respectively. And therefore, we find for the input reduction target and the output augmentation target the following expressions:

input reduction target:

$$\sum_m v_m^*(x_{mo} - d_{mo}^x) = 1 - (1 - \theta^*) \times \frac{1}{(1+\theta^*)} = \frac{2\theta^*}{1+\theta^*} \tag{2.8}$$

output augmentation target:

$$\sum_s u_s^*(y_{so} + d_{so}^y) = \theta^* + (1 - \theta^*) \times \frac{\theta^*}{(1+\theta^*)} = \frac{2\theta^*}{1+\theta^*} \tag{2.9}$$

An illustration of these improvement functions is given in Fig. 3.

It is now possible to determine each optimal distance d_{mo}^{x*} and d_{so}^{y*} by using the MOQP model (2.1)–(2.7).

The friction minimization solution for an inefficient DMU_o can next be expressed by means of (2.10) and (2.11):

$$x_{mo}^* = x_{mo} - d_{mo}^{x*} \tag{2.10}$$

$$y_{so}^* = y_{so} + d_{so}^{y*}. \tag{2.11}$$

By means of the DFM model, it is possible to present a new efficiency-improvement solution based on the standard CCR projection. This means an increase in new options for efficiency-improvement solutions in DEA. The main advantage of the DFM model is that it yields an outcome on the efficient frontier that is as close as possible to the DMU's input and output profile (see Fig. 4), which means that the DFM projection can compute more effectively an appropriate solution than the approach in the standard CCR projection model.

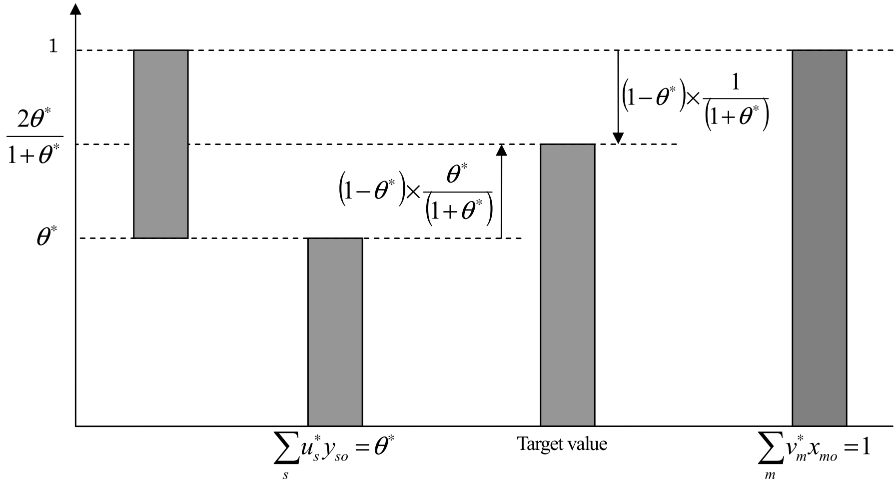


Fig. 3 Presentation of balanced allocation for the total efficiency gap

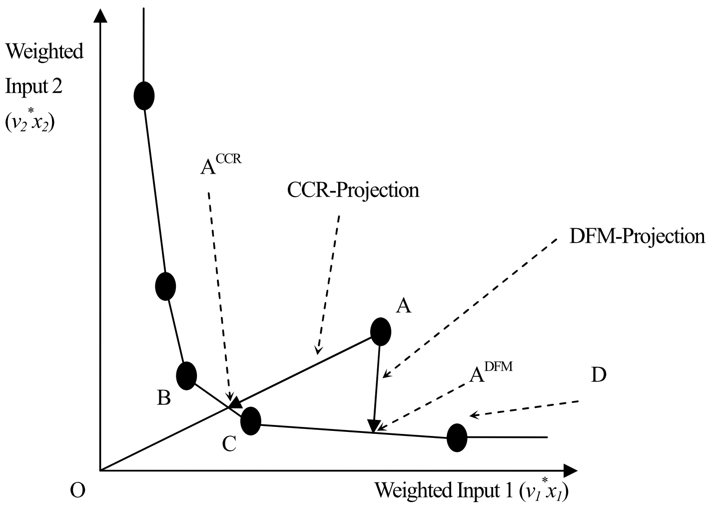


Fig. 4 Degree of improvement of a DFM-projection and a CCR-projection in weighted input space

3 Stepwise-DFM model in DEA

3.1 Outline of context-dependent model

The Context-Dependent (CD, hereafter) model can generate efficient frontiers in successive levels, and can yield a stepwise level-by-level improvement projection (see Seiford and Zhu 2003). The CD model is formulated below.

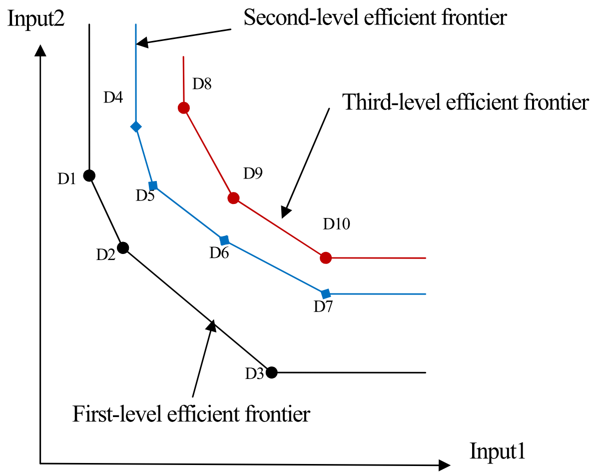


Fig. 5 Illustration of the CD model

Let $J^l = \{DMU_j, j = 1, \dots, J\}$ be the set of all J DMUs. We interactively define $J^{l+1} = J^l - E^l$ where $E^l = \{DMU_k \in J^l | \theta^*(l,k)\} = 1$ and $\theta^*(l,k)$ is the optimal value by using formula (A.1).

When $l = 1$, the model becomes the original CCR model, while the DMUs in set E_1 define the first-level efficient frontier. When $l = 2$, it gives the second-level efficient frontier after the exclusion of the first-level efficient DMUs. And so on. In this manner, we identify several levels of efficient frontiers. We call El the lth-level efficient frontier. The following algorithm accomplishes the identification of these efficient frontiers.

Step 1: Set $l = 1$. Evaluate the entire set of DMUs, J_1. We obtain then the first-level efficient DMUs for set E_1 (the first-level efficient frontier).

Step 2: Exclude the efficient DMUs from future DEA runs. $J^{l+1} = J^l - E^l$. (If $J^{l+1} = \phi$, then stop.)

Step 3: Evaluate the new subset of "inefficient" DMUs. We obtain then a new set of efficient DMUs E^{l+1} (the new efficient frontier).

Step 4: Let $l = l + 1$. Go to step 2.

Stopping rule: $J^{l+1} = \phi$, the algorithm is terminated.

A visual presentation of the CD model is given in Fig. 5.

3.2 Proposal of Stepwise-DFM model

We will not try to merge the merits of the DFM and the CD approach. To that end, we will design a Stepwise DFM model that is integrated with a DFM and CD model.

Any efficiency-improving projection model which includes the standard CCR projection supplemented with the DFM-projection is always directed towards achieving "full efficiency". This strict condition may not always be easy to achieve in reality. Therefore, in this section we will develop a new efficiency improving projection model, which aims to integrate with CD model and DFM approach, coined the "Stepwise Distance Friction Minimization" (Stepwise DFM hereafter) model. It can yield a stepwise efficiency improving projection that depends on l-level efficient frontiers (l-level DFM projection), as shown in Fig. 5.

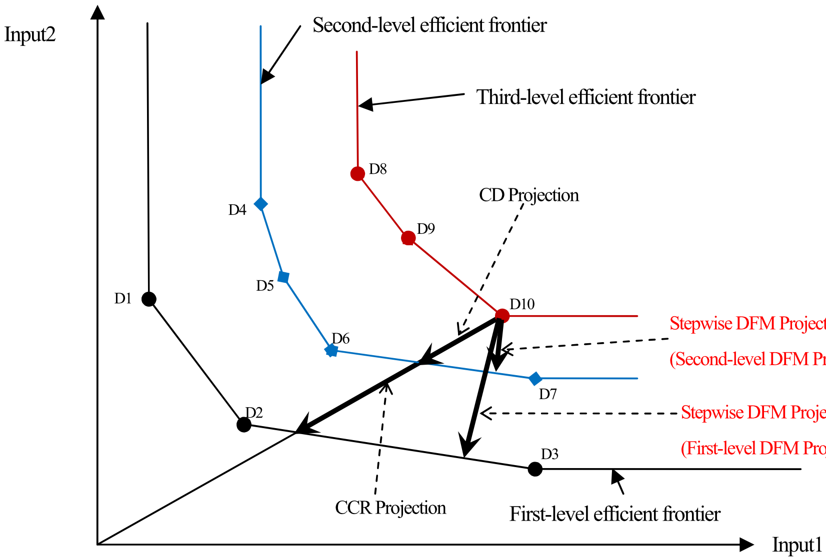


Fig. 6 Illustration of the CD model

For example, a second-level DFM projection for DMU10 (D10) aims to position DMU10 on a second-level efficient frontier. And a first-level DFM projection is just equal to a DFM projection (2.1)–(2.7). We notice here that the second-level DFM projection is easier to achieve than a first-level DFM projection. A stepwise-DFM model can yield a more practical and realistic efficiency improving projection than a CCR projection or a DFM projection.

The advantage of the Stepwise DFM model is also that it yields an outcome on a l-level efficient frontier that is as close as possible to the DMU's input and output profile, which means that the Stepwise DFM projection can compute more effective solutions than the CD projection model (see Fig. 6). The operational character of this model will now be tested in Sect. 4.

4 Application of a stepwise DFM model to public transport efficiency management in Japan

4.1 Database and analysis framework

The stepwise DFM DEA model offers many opportunities for a critical comparative judgment of the performance of corporate organizations in both the public and private sector. As an empirical illustration, we will offer here a benchmark analysis of the efficiency achievements of Japanese public transport companies. In our empirical work, we use input and output data for a set of 9 urban transportation authorities and 16 major private railway companies in Japan. The DMUs used in our analysis are listed in Table 1.

Table 1 A listing of DMUs

No	Major private railway companies	No	Urban transportation authorities
1	Tobu	1	Sapporo
2	Seibu	2	Sendai
3	Keisei	3	Tokyo
4	Keio	4	Yokohama
5	Odakyu	5	Nagoya
6	Tokyu	6	Kyoto
7	Keikyu	7	Osaka
8	Sotetsu	8	Kobe
9	Meitetsu	9	Fukuoka
10	Kintetsu		
11	Nankai		
12	Keihan		
13	Hankyu		
14	Hanshin		
15	Nishitetsu		
16	Tokyometro		

In this study we use the following inputs and outputs:

- Input:
  - (I) Operating cost (in 2007);
  - (I) Railway business property (in 2007);
- Output:
  - (O) Operating revenues (in 2007);

All data were obtained from the "Railway annual statement 2007". In our application, we first applied the standard CCR model, while next the results were used to determine the CCR and DFM projections. Additionally, we applied the CD model, and then the results were used to determine the CD and Stepwise DFM projections. Finally, these various results were mutually compared.

4.2 Efficiency evaluation based on the CCR model

The efficiency evaluation results for the 25 public transport corporations based on the CCR model is given in Fig. 7. From Fig. 7, it can be seen that Keio and Tokyometro are efficiently operating firms. On the other hand, Kyoto has a low efficiency (i.e., an efficiency score around 50 per cent). Furthermore, Kobe and Fukuoka also have a low efficiency.

It is noteworthy that the average efficiency level of urban transportation authorities is relatively low compared to major private railway companies. It seems thus plausible that apparently transportation authorities have still much room for further efficiently-enhancing strategies.

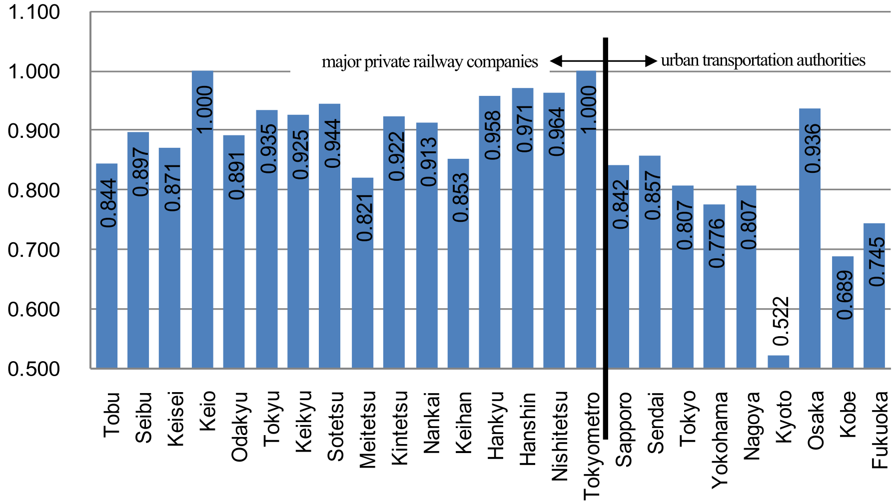


Fig. 7 Efficiency scores based on the CCR model

4.3 Direct efficiency improvement projection based on the CCR and DFM models

The direct efficiency improvement projection results based on the CCR and DFM model for inefficient public transport corporations are presented in Table 2.

In Table 2, it appears that the empirical ratios of change in the DFM projection are smaller than those in the CCR projection, as was to be expected. In Table 2, this applies particularly to Seibu, Tokyu, Keikyu, Hanshin and Nishitetsu, which are apparently non-slack type (i.e., s^{-**} and s^{+**} are zero) corporations. The DFM projection involves both an input reduction and an output increase, and, clearly, the DFM projection does not involve a uniform ratio, because this model looks for the optimal input reduction (i.e., the shortest distance to the frontier, or distance friction minimization). For instance, the CCR projection shows that Seibu in Table 2 should reduce the Operating cost and the Railway business property by 10.34 per cent in order to become efficient. On the other hand, the DFM results show that a reduction in Railway business property of 9.96 per cent and an increase in the Operating revenues of 5.45 per cent are required to become efficient. Apart from the practicality of such a solution, the models show clearly that a different—and perhaps more effective—solution is available than the standard CCR projection to reach the efficiency frontier.

4.4 Stepwise efficiency improvement projection based on the CD and stepwise DFM models

The efficiency improvement projection results for the nearest upper level efficient frontier based on the CD and Stepwise-DFM model for inefficient public transport corporation are presented in Table 3.

It appears that the ratios of change in the Stepwise DFM projection are smaller than those in the CD projection, as was expected. In Table 3, this particularly applies

Table 2 Direct efficiency-improvement projection results of the CCR and DFM model

DMU	Score (θ^*)	CCR projection			DFM projection				
		Score (θ^{**})			Score (θ^{**})				
I/O	Data	Projection	Difference	%			Projection	Difference	%
					d_{mo}^{x*}	s^{-**}	x_{mo}^{**}	$d_{mo}^{x*}+s^{-**}$	
					d_{so}^{y*}	s^{+**}	y_{so}^{**}	$d_{so}^{y*}+s^{+**}$	
Tobu	0.844	1.000			1.000				
(I) cost	137242584	115874608	−21367976	−15.57%	−11585923	0	125656661	−11585923	−8.44%
(I) property	712422107	475479060	−236943047	−33.26%	0	−196803438	515618669	−196803438	−27.62%
(O) revenue	160818200	160818200	0	0.00%	13576160	0	174394360	13576160	8.44%
Seibu	0.897	1.000			1.000				
(I) cost	84550368	75806930	−8743438	−10.34%	0	0	84550368	0	0.00%
(I) property	329209999	295166066	−34043933	−10.34%	−32801294	0	296408705	−32801294	−9.96%
(O) revenue	102197169	102197169	0	0.00%	5572273	0	107769442	5572273	5.45%
Keisei	0.871	1.000			1.000				
(I) cost	45143268	39338162	−5805106	−12.86%	−3102001	0	42041267	−3102001	−6.87%
(I) property	203714344	161419940	−42294404	−20.76%	0	−31202500	172511844	−31202500	−15.32%
(O) revenue	54596020	54596020	0	0.00%	3751543	0	58347563	3751543	6.87%
Odakyu	0.891	1.000			1.000				
(I) cost	95105070	84733876	−10371194	−10.90%	−5484647	0	89620423	−5484647	−5.77%
(I) property	503547659	347696396	−155851263	−30.95%	0	−135799840	367747819	−135799840	−26.97%
(O) revenue	117599098	117599098	0	0.00%	6781863	0	124380961	6781863	5.77%
Tokyu	0.935	1.000			1.000				
(I) cost	116330884	108801281	−7529603	−6.47%	0	0	0	0	0.00%
(I) property	448779376	419731796	−29047580	−6.47%	−27543332	0	−55086663	−27543332	−6.14%
(O) revenue	145938161	145938161	0	0.00%	4880939	0	9761879	4880939	3.34%
Keikyu	0.925	1.000			1.000				
(I) cost	64879034	60022099	−4856935	−7.49%	0	0	0	0	0.00%
(I) property	240695337	222676548	−18018789	−7.49%	−17487164	0	−34974329	−17487164	−7.27%
(O) revenue	78827586	78827586	0	0.00%	3065308	0	6130617	3065308	3.89%
Sotetsu	0.944	1.000			1.000				
(I) cost	26015702	24568725	−1446977	−5.56%	−744184	0	−1488369	−744184	−2.86%
(I) property	111527822	100815133	−10712689	−9.61%	0	−7828852	−7828852	−7828852	−7.02%
(O) revenue	34098049	34098049	0	0.00%	975381	0	1950763	975381	2.86%
Meitetsu	0.821	1.000			1.000				
(I) cost	76843610	63078192	−13765418	−17.91%	−7559826	0	−15119652	−7559826	−9.84%
(I) property	409977161	258834612	−151142549	−36.87%	0	−125678563	−125678563	−125678563	−30.66%
(O) revenue	87543953	87543953	0	0.00%	8612519	0	17225038	8612519	9.84%
Kintetsu	0.922	1.000			1.000				
(I) cost	131011669	120851064	−10160605	−7.76%	−5285251	0	−10570503	−5285251	−4.03%
(I) property	771942168	495899414	−276042754	−35.76%	0	−256037261	−256037261	−256037261	−33.17%
(O) revenue	167724844	167724844	0	0.00%	6766328	0	13532657	6766328	4.03%
Nankai	0.913	1.000			1.000				
(I) cost	46384894	42356020	−4028874	−8.69%	−2105893	0	−4211786	−2105893	−4.54%
(I) property	294000567	173803399	−120197168	−40.88%	0	−112306423	−112306423	−112306423	−38.20%
(O) revenue	58784397	58784397	0	0.00%	2668836	0	5337671	2668836	4.54%
Keihan	0.853	1.000			1.000				
(I) cost	46034077	39281757	−6752320	−14.67%	−3643366	0	−7286733	−3643366	−7.91%
(I) property	199915154	161188487	−38726667	−19.37%	0	−25969407	−25969407	−25969407	−12.99%
(O) revenue	54517737	54517737	0	0.00%	4314805	0	8629611	4314805	7.91%
Hankyu	0.958	1.000			1.000				
(I) cost	75171681	72005545	−3166136	−4.21%	−1617123	0	−3234247	−1617123	−2.15%
(I) property	399741850	295467053	−104274797	−26.09%	0	−97918591	−97918591	−97918591	−24.50%
(O) revenue	99933906	99933906	0	0.00%	2149818	0	4299637	2149818	2.15%

Table 2 (*Continued*)

DMU I/O	Score (θ^*) Data	CCR projection Score (θ^{**}) Projection	Difference	%	DFM projection Score (θ^{**}) d^{x*}_{mo} d^{y*}_{so}	s^{-**} s^{+**}	Projection x^{**}_{mo} y^{**}_{so}	Difference $d^{x*}_{mo}+s^{-**}$ $d^{y*}_{so}+s^{+**}$	%
Hanshin	0.971	1.000			1.000				
(I) cost	20880360	20265374	−614986	−2.95%	0	0	0	0	0.00%
(I) property	71623305	69513796	−2109509	−2.95%	−2075902	0	−4151803	−2075902	−2.90%
(O) revenue	25540600	25540600	0	0.00%	381743	0	763486	381743	1.49%
Nishitetsu	0.964	1.000			1.000				
(I) cost	18416583	17754279	−662304	−3.60%	0	0	0	0	0.00%
(I) property	66379457	63992294	−2387163	−3.60%	−2301763	0	−4603526	−2301763	−3.47%
(O) revenue	22961699	22961699	0	0.00%	420439	0	840877	420439	1.83%
Sapporo	0.842		1.000			1.000			
(I) cost	31887493	26834609	−5052884	−15.85%	−2743836	0	−5487671	−2743836	−8.60%
(I) property	406895116	110112949	−296782167	−72.94%	0	−287307235	−287307235	−287307235	−70.61%
(O) revenue	37242789	37242789	0	0.00%	3204645	0	6409290	3204645	8.60%
Sendai	0.857	1.000			1.000				
(I) cost	9547699	8182994	−1364705	−14.29%	−734872	0	−1469744	−734872	−7.70%
(I) property	123357198	33578041	−89779157	−72.78%	0	−87194706	−87194706	−87194706	−70.68%
(O) revenue	11356883	11356883	0	0.00%	874122	0	1748245	874122	7.70%
Tokyo	0.807		1.000			1.000			
(I) cost	112204498	90536745	−21667753	−19.31%	−11991735	0	−23983470	−11991735	−10.69%
(I) property	1692909251	371507851	−1321401400	−78.06%	0	−1281696898	−1281696898	−1281696898	−75.71%
(O) revenue	125652692	125652692	0	0.00%	13428996	0	26857993	13428996	10.69%
Yokohama	0.776 7	1.000			1.000				
(I) cost	28808045	22360376	−6447669	−22.38%	−3630066	0	−7260131	−3630066	−12.60%
(I) property	735299032	91753413	−643545619	−87.52%	0	−631983887	−631983887	−631983887	−85.95%
(O) revenue	31033162	31033162	0	0.00%	3910450	0	7820900	3910450	12.60%
Nagoya	0.807	1.000			1.000				
(I) cost	61326002	49516496	−11809506	−19.26%	−6533864	0	−13067727	−6533864	−10.65%
(I) property	780732042	203185646	−577546396	−73.97%	0	−555898363	−555898363	−555898363	−71.20%
(O) revenue	68722164	68722164	0	0.00%	7321874	0	14643748	7321874	10.65%
Kyoto	0.522	1.000			1.000				
(I) cost	29271536	15273060	−13998476	−47.82%	−9198802	0	−18397604	−9198802	−31.43%
(I) property	494381778	62671366	−431710412	−87.32%	0	−412015460	−412015460	−412015460	−83.34%
(O) revenue	21196930	21196930	0	0.00%	6661296	0	13322591	6661296	31.43%
Osaka	0.936	1.000			1.000				
(I) cost	117496019	109938219	−7557800	−6.43%	−3904476	0	−7808951	−3904476	−3.32%
(I) property	1248374651	451119722	−797254929	−63.86%	0	−782263903	−782263903	−782263903	−62.66%
(O) revenue	152579299	152579299	0	0.00%	5070318	0	10140635	5070318	3.32%
Kobe	0.689	1.000			1.000				
(I) cost	18685348	12881804	−5803544	−31.06%	−3435255	0	−6870511	−3435255	−18.38%
(I) property	309292607	52859107	−256433500	−82.91%	0	−246715483	−246715483	−246715483	−79.77%
(O) revenue	17878193	17878193	0	0.00%	3286862	0	6573724	3286862	18.38%
Fukuoka	0.745	1.000			1.000				
(I) cost	22083430	16453495	−5629935	−25.49%	−3226212	0	−6452423	−3226212	−14.61%
(I) property	491943185	67515157	−424428028	−86.28%	0	−414564606	−414564606	−414564606	−84.27%
(O) revenue	22835214	22835214	0	0.00%	3336041	0	6672082	3336041	14.61%

Table 3 Efficiency-improvement projection results for nearest upper level efficient frontier

	DMU	Score (θ^*)	CD projection		Stepwise-DFM projection	
	I/O	Data	Difference	%	Difference $d^{x*}_{mo} + s^{-**}$ $d^{y*}_{so} + s^{+**}$	%
E2	Sotetsu	0.944				
	(I) cost	26015702	−1446977	−5.56%	−744184	−2.86%
	(I) property	111527822	−10712689	−9.61%	−7828852	−7.02%
	(O) revenue	34098049	0	0.00%	975381	2.86%
	Hankyu	0.958				
	(I) cost	75171681	−3166136	−4.21%	−1617123	−2.15%
	(I) property	399741850	−104274797	−26.09%	−97918591	−24.50%
	(O) revenue	99933906	0	0.00%	2149818	2.15%
	Hanshin	0.971				
	(I) cost	20880360	−614986	−2.95%	0	0.00%
	(I) property	71623305	−2109509	−2.95%	−2075902	−2.90%
	(O) revenue	25540600	0	0.00%	381743	1.49%
	Nishitetsu	0.964				
	(I) cost	18416583	−662304	−3.60%	0	0.00%
	(I) property	66379457	−2387163	−3.60%	−2301763	−3.47%
	(O) revenue	22961699	0	0.00%	420439	1.83%
E3	Tokyo	0.987				
	(I) cost	116330884	−1465276	−1.26%	−1029922	−0.89%
	(I) property	448779376	−5652717	−1.26%	0	0.00%
	(O) revenue	145938161	0	0.00%	924926	0.63%
	Keikyu	0.967				
	(I) cost	64879034	−2151905	−3.32%	−1511718	−2.33%
	(I) property	240695337	−7983371	−3.32%	0	0.00%
	(O) revenue	78827586	0	0.00%	1329320	1.69%
	Kintetsu	0.963				
	(I) cost	131011669	−4846697	−3.70%	−2469018	−1.88%
	(I) property	771942168	−101032343	−13.09%	−88388517	−11.45%
	(O) revenue	167724844	0	0.00%	3160907	1.88%
	Osaka	0.977				
	(I) cost	117496019	−2723737	−2.32%	−1377839	−1.17%
	(I) property	1248374651	−638047949	−51.11%	−630890840	−50.54%
	(O) revenue	152579299	0	0.00%	1789250	1.17%
E4	Seibu	0.963				
	(I) cost	84550368	−3115939	−3.69%	−1652015	−1.95%
	(I) property	329209999	−12132392	−3.69%	0	0.00%
	(O) revenue	102197169	0	0.00%	1918490	1.88%
	Nankai	0.989				
	(I) cost	46384894	−529772	−1.14%	−271618	−0.59%
	(I) property	294000567	−3357848	−1.14%	0	0.00%
	(O) revenue	58784397	0	0.00%	337623	0.57%

Table 3 (*Continued*)

	DMU	Score (θ^*)	CD projection		Stepwise-DFM projection	
	I/O	Data	Difference	%	Difference $d_{mo}^{x*}+s^{-**}$ $d_{so}^{y*}+s^{+**}$	%
E5	Keisei	0.988				
	(I) cost	45143268	−522332	−1.16%	−288164	−0.64%
	(I) property	203714344	−2357087	−1.16%	0	0.00%
	(O) revenue	54596020	0	0.00%	317691	0.58%
	Odakyu	0.995				
	(I) cost	95105070	−442591	−0.47%	−247053	−0.26%
	(I) property	503547659	−2343361	−0.47%	0	0.00%
	(O) revenue	117599098	0	0.00%	274274	0.23%
	Keihan	0.971				
	(I) cost	46034077	−1328346	−2.89%	−736796	−1.60%
	(I) property	199915154	−5768692	−2.89%	0	0.00%
	(O) revenue	54517737	0	0.00%	798088	1.46%
E6	Tobu	0.950				
	(I) cost	137242584	−6805930	−4.96%	−4086177	−2.98%
	(I) property	712422107	−35329378	−4.96%	0	0.00%
	(O) revenue	160818200	0	0.00%	4088914	2.54%
	Sendai	0.962				
	(I) cost	9547699	−363129	−3.80%	−185084	−1.94%
	(I) property	123357198	−74728153	−60.58%	−73785469	−59.81%
	(O) revenue	11356883	0	0.00%	220156	1.94%
E7	Meitetsu	0.972				
	(I) cost	76843610	−2154485	−2.80%	−1104073	−1.44%
	(I) property	409977161	−11494638	−2.80%	0	0.00%
	(O) revenue	87543953	0	0.00%	1244695	1.42%
	Sapporo	0.982				
	(I) cost	31887493	−567949	−1.78%	−293748	−0.92%
	(I) property	406895116	−7247223	−1.78%	0	0.00%
	(O) revenue	37242789	0	0.00%	334647	0.90%
E8	Nagoya	0.960				
	(I) cost	61326002	−2479943	−4.04%	−1321222	−2.15%
	(I) property	780732042	−31571779	−4.04%	0	0.00%
	(O) revenue	68722164	0	0.00%	1418192	2.06%
E9	Tokyo	0.999				
	(I) cost	112204498	−75066	−0.07%	−37545	−0.03%
	(I) property	1692909251	−265406432	−15.68%	−264928768	−15.65%
	(O) revenue	125652692	0	0.00%	42045	0.03%
E10	Yokohama	0.962				
	(I) cost	28808045	−1096260	−3.81%	−558762	−1.94%
	(I) property	735299032	−317191579	−43.14%	−309081955	−42.03%
	(O) revenue	31033162	0	0.00%	601920	1.94%

Table 3 (*Continued*)

	DMU	Score (θ^*)	CD projection		Stepwise-DFM projection	
	I/O	Data	Difference	%	Difference $d_{mo}^{x*} + s^{-**}$ $d_{so}^{y*} + s^{+**}$	%
	Kobe	0.854				
	(I) cost	18685348	−2720599	−14.56%	−1467105	−7.85%
	(I) property	309292607	−68421060	−22.12%	−49508705	−16.01%
	(O) revenue	17878193	0	0.00%	1403730	7.85%
	Fukuoka	0.923				
	(I) cost	22083430	−1692194	−7.66%	−879805	−3.98%
	(I) property	491943185	−184286067	−37.46%	−172028986	−34.97%
	(O) revenue	22835214	0	0.00%	909757	3.98%
E11	Kyoto	0.753				
	(I) cost	29271536	−7222361	−24.67%	−5399466	−18.45%
	(I) property	494381778	−121982117	−24.67%	0	0.00%
	(O) revenue	21196930	0	0.00%	2983043	14.07%

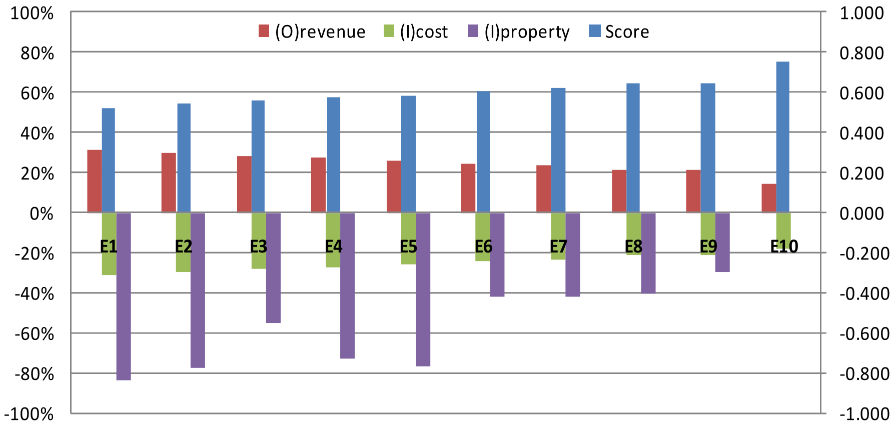


Fig. 8 Efficiency improvement projection results based on the Stepwise-DFM model (Kyoto)

to Tobu, Seibu, Keisei, Odakyu, Tokyu, Keikyu, Meitetsu, Nankai, Heihan, Hanshin, Nishitetsu, Sapporo, Nagoya, and Kyoto, which are non-slack type (i.e. s^{-**} and s^{+**} are zero) corporations. Again, the results of our model applications show clearly that a different—and perhaps more effective—solution may exist than the CD projection.

The Stepwise-DFM model appears to be able to present a more realistic efficiency-improvement plan, if we compare the results of Tables 2 and 3. For instance, the DFM results in Table 2 show that Fukuoka should reduce the Operating cost by 14.61 per cent and the Railway business property by 84.27 per cent, and increase the Operating revenues of 14.61 per cent in order to become efficient. On the other

Table 4 Stepwise-efficiency improvement projection results for all level efficient frontier of Kyoto City

DMU	Score (θ^*)	CD projection	CD-DFM projection	DMU	Score (θ^*)	CD projection	CD-DFM projection
I/O	Data	%	%	I/O	Data	%	%
E1	0.522			E6	0.609		
(I) cost	29271536	−47.82%	−31.43%	(I) cost	29271536	−39.12%	−24.32%
(I) property	494381778	−87.32%	−83.34%	(I) property	494381778	−53.43%	−42.10%
(O) revenue	21196930	0.00%	31.43%	(O) revenue	21196930	0.00%	24.32%
E2	0.545			E7	0.620		
(I) cost	29271536	−45.53%	−29.47%	(I) cost	29271536	−38.00%	−23.46%
(I) property	494381778	−82.85%	−77.79%	(I) property	494381778	−53.16%	−42.17%
(O) revenue	21196930	0.00%	29.47%	(O) revenue	21196930	0.00%	23.46%
E3	0.558			E8	0.646		
(I) cost	29271536	−44.24%	−28.40%	(I) cost	29271536	−35.38%	−21.49%
(I) property	494381778	−64.92%	−54.96%	(I) property	494381778	−51.29%	−40.82%
(O) revenue	21196930	0.00%	28.40%	(O) revenue	21196930	0.00%	21.49%
E4	0.571			E9	0.647		
(I) cost	29271536	−42.86%	−27.27%	(I) cost	29271536	−35.34%	−21.46%
(I) property	494381778	−78.56%	−72.71%	(I) property	494381778	−42.23%	−29.84%
(O) revenue	21196930	0.00%	27.27%	(O) revenue	21196930	0.00%	21.46%
E5	0.586			E10	0.753		
(I) cost	29271536	−41.44%	−26.13%	(I) cost	29271536	−24.67%	−18.45%
(I) property	494381778	−81.64%	−76.84%	(I) property	494381778	−24.67%	0.00%
(O) revenue	21196930	0.00%	26.13%	(O) revenue	21196930	0.00%	14.07%

hand, the Stepwise DFM results in Table 3 show that a reduction in Operating cost of 3.98 per cent and Railway business property of 34.97 per cent, and an increase in the Operating revenues of 3.98 per cent are required to become efficient. The Stepwise DFM model provides the policy decision-maker with more flexible, practical and transparent solutions that are available in the DFM projection to reach the nearest upper level efficiency frontier.

Finally, the stepwise efficiency improvement projection results for all level efficient frontiers of Kyoto (lowest efficiency level DMU; E11) based on the CD and Stepwise-DFM model are presented in Table 4, while a comparative result of the stepwise DFM model for Kyoto is presented in Fig. 8.

The findings from Fig. 8 illustrate, for instance, that, if the Kyoto city wishes to implement an efficiency improvement plan with a E10 level, only a reduction in the operating cost of 18.45 per cent and an increase in operating revenue of 14.07 per cent are required, while then the efficiency level rises to the E10 level efficient frontier.

These results offer a meaningful contribution to decision support and planning for the efficiency improvement of public transport operations. In conclusion, this Stepwise DFM model may become a policy vehicle that may have great added value for decision making and planning of both public and private actors.

5 Conclusion

In this paper we have presented a new methodology, the Stepwise DFM model, which is integrated with a DFM and CD model. This new methodology does not require a uniform reduction of all inputs, as in the standard model. Instead, the new method minimizes the distance friction for each input and output separately. As a result, the reductions in inputs and increases in outputs do necessarily reach an efficiency frontier that is smaller than in the standard model. This offers more flexibility for the operational management of an organization. In addition, the stepwise projection allows DMUs to include various levels of ambition regarding the ultimate performance in their strategic judgment. In conclusion, our Stepwise DFM model is able to present a more realistic efficiency-improvement plan, and may thus provide a meaningful contribution to decision making and planning for efficiency improvement of relevant agents.

Appendix: Efficiency improvement projection in DEA: the standard approach

The standard Charnes et al. (1978) model (abbreviated hereafter as the CCR-input model) for a given DMU$_j$ $(j = 1, \dots, J)$ to be evaluated in any trial o (where o ranges over $1, 2, \dots, J$) may be represented as the following fractional programming (FP_o) problem:

$$\begin{aligned} (FP_o) \quad & \max_{v,u} \ \theta = \frac{\sum_s u_s y_{so}}{\sum_m v_m x_{mo}} \\ & \text{s.t.} \quad \frac{\sum_s u_s y_{sj}}{\sum_m v_m x_{mj}} \le 1 \quad (j = 1, \dots, J) \\ & v_m \ge 0, \qquad u_s \ge 0, \end{aligned} \tag{A.1}$$

where θ represents an objective variable function (efficiency score); x_{mj} is the volume of input m $(m = 1, \dots, M)$ for DMU j $(j = 1, \dots, J)$; y_{sj} is the output s $(s = 1, \dots, S)$ of DMU j; and v_m and u_s are the weights given to input m and output s, respectively. Model (A.1) is often called an input-oriented CCR model, while its reciprocal (i.e. an interchange of the numerator and denominator in objective function (A.1), with a specification as a minimization problem under an appropriate adjustment of the constraints) is usually known as an output-oriented CCR model. Model (A.1) is obviously a fractional programming model, which may be solved stepwise by first assigning an arbitrary value to the denominator in (A.1), and then maximizing the numerator.

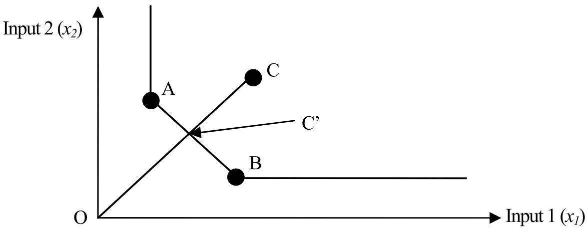
The improvement projection $(\hat{x}_o, \hat{y}_o)$ can now be defined in (A.2) and (A.3) as:

$$\hat{x}_o = \theta^* x_o - s^{-*}; \tag{A.2}$$

$$\hat{y}_o = y_o + s^{+*}. \tag{A.3}$$

These equations indicate that the efficiency of (x_o, y_o) for DMU$_\text{o}$ can be improved if the input values are reduced radially by the ratio θ^*, and the input excesses s^{-*} are

Fig. 9 Illustration of original DEA projection in input space



eliminated (see Fig. 9). The original DEA models presented in the literature have thus far only focused on a uniform input reduction or a uniform output increase in the efficiency-improvement projections, as shown in Fig. 1 ($\theta^* = \mathrm{OC'}/\mathrm{OC}$).

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